
MATHEMATICS ESSAY PROBLEMS

Name : _____

Participant ID : _____

Country : _____



21st International Mathematics and Science Olympiad
Wenzhou, China
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Instructions:

1. Write your name, country and Participant ID on every page of this booklet.
2. Write your detailed solution in Arabic Numerical or English in the space provided for each question in this booklet. If you need more space for your working solutions, you may use the reverse side of each page, but please write “next side continued” in the last line.
3. There are 13 questions in this paper.
4. Each question is worth 3 marks and partial credits may be awarded.
5. Diagrams are NOT drawn to scale. They are intended only as aids.
6. You have 90 minutes to complete this paper.
7. Use black pen or blue pen or pencil to write your answer.

Do not turn over this page until you are told to do so.

ESSAY PROBLEMS

Name: _____ Country: _____ Participant ID: _____

1. Xiaoming writes down all the perfect square numbers continuously, starting from 1, in increasing order from left to right, one after the other as shown:

1491625364964...

Which digit is located in the 121st place starting from the left?

Answer: _____

ESSAY PROBLEMS

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2. How many isosceles triangles are there which all side lengths are integers less than 10?

Answer: _____

ESSAY PROBLEMS

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3. A four-digit number is said to be *nice* if it satisfies the following conditions:

- all the digits are distinct,
- the number is divisible by 6,
- the digits are in descending order from left to right,
- the number is greater than 9000 but less than 10000.

How many *nice* numbers are there?

Answer: _____

ESSAY PROBLEMS

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4. Tom and Jerry play a game. They alternately take some marble(s) out of a box containing 2024 marbles, starting with Tom. In each turn, each player must take 2^m marble(s) from the box, where m is a non-negative integer, i.e., $m = 0, 1, 2, 3, \dots$. For example, a player may take 1 marble, 4 marbles or 32 marbles, but cannot take 10 marbles, because 10 is not a power of 2. The player who takes the last marble(s) from the box wins the game. Determine who has a winning strategy and explain the winning strategy.

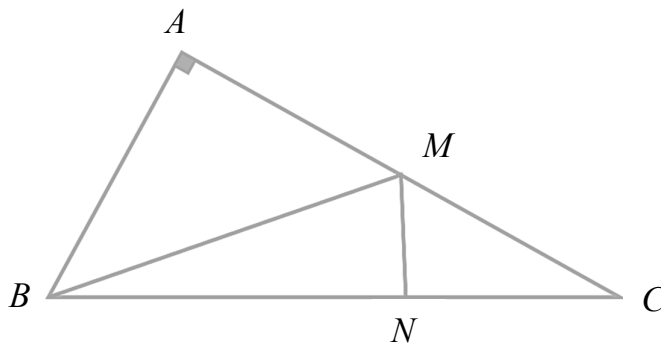
Answer: _____

ESSAY PROBLEMS

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5. In the diagram, ABC is a right-angled triangle at A , such that $AB = 8$ cm, $AC = 12$ cm and M is the midpoint of AC . Let N be a point on BC such that $BN = 2NC$.

What is the length, in cm, of MN ?



Answer: _____

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6. Nala has six cards. All the numbers on the cards are consecutive two-digit factors of the product $2023 \times 2024 \times 2025$. He selects the minimum number of cards where the sum of all digits is 21. He arranges all the selected cards in a line to form a bigger number. What is the second largest number that Nala can form?

Answer: _____

ESSAY PROBLEMS

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7. Tony has two water tanks, each has a capacity of 1234 liters. At the beginning, Tank Red is full of water, while Tank Blue is empty. Tony accepts the challenge from his friend that for each turn, he can do one of the following options:

- *Option A:* Pour 1 liter of water from Tank Red to Tank Blue,
- *Option B:* Drain out the water from Tank Red out with the same volume as the volume of water in Tank Blue at that time.

What is the minimum number of turns that Tony takes such that the amount of water in Tank Red will be the same as that in Tank Blue?

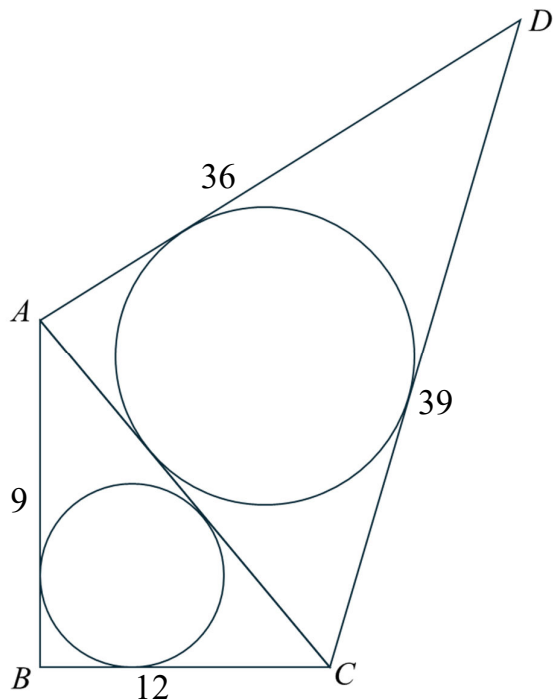
Answer: _____

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8. In the diagram, $ABCD$ is a quadrilateral where $\angle ABC = 90^\circ$, $AB = 9$, $BC = 12$, $CD = 39$ and $DA = 36$. Circle M is tangent to each side of $\triangle ABC$, and circle N is tangent to each side of $\triangle ACD$.

What is the ratio of the radius of circle N to the radius of circle M ?



Answer: _____

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9. A 5×5 table is called a *UCAS table* if it satisfies the following conditions:

- Each cell is coloured with grey or white,
- Any two adjacent cells (sharing an edge) must have different colours,
- The 25 cells are filled with 25 consecutive integers, which must include 2024,
- The numbers in any two adjacent cells (sharing an edge) must have different parity (cannot be both odd or both even).

Now, your tasks are to:

- Construct a *UCAS table* which gives the greatest possible sum of all integers in the shaded cells. What is the sum?
- Construct a *UCAS table* which gives the smallest possible sum of all integers in the shaded cells. What is the sum?

a.

b.

Answer:

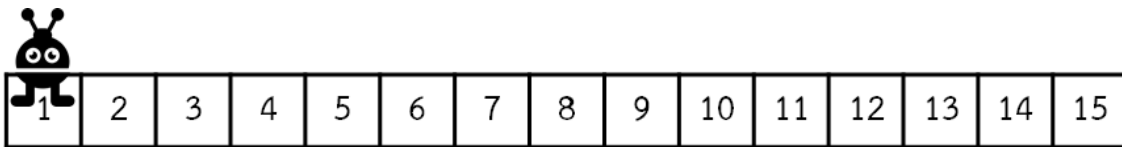
greatest sum = _____

smallest sum = _____

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10. The *IMSO ROBOT* stands at box 1 in a path with 15 boxes that are arranged in a line as shown in the diagram below. In each turn, the *IMSO ROBOT* can only move 1, 2 or 3 step(s) to the right. So, in the first turn, the *IMSO ROBOT* can either move one step to land into box 2, two steps to land into box 3 or three steps to land into box 4. If the *IMSO ROBOT* cannot land outside of the boxes, what is the total number of ways to move the *IMSO ROBOT* to land into box 15?



Answer: _____

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11. Let $S = 1^1 + 2^2 + 3^3 + 4^4 + \dots + 2023^{2023} + 2024^{2024}$.

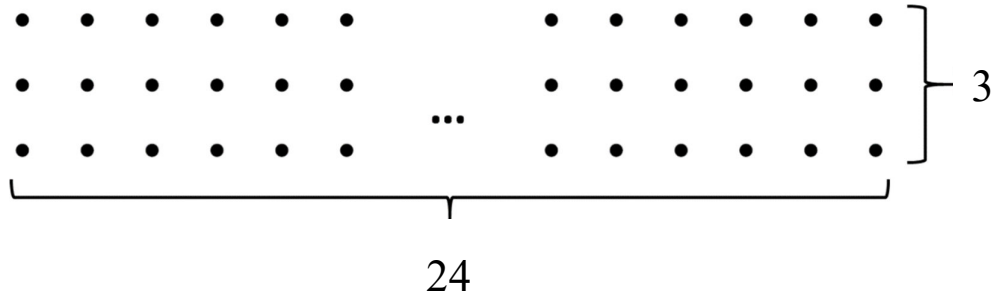
What is the unit digit of S ?

Answer: _____

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12. In the diagram, 72 dots lie in 3 rows and 24 columns. The distance between consecutive rows and the consecutive columns are equal. In how many ways can we draw a straight line passing through at least 3 dots?



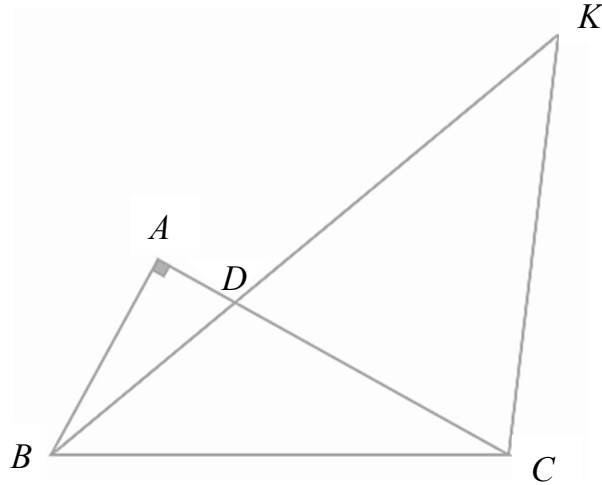
Answer: _____

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13. In the diagram, ABC is a right-angled triangle at A and $\angle ABC = 60^\circ$.

On side AC , point D is taken such that $\angle ABD = 20^\circ$. On ray BD , point K is taken such that $DK = BC$. What is the measure, in degrees, of $\angle KCB$?



Answer: _____