MATHEMATICS
SHORT ANSWER PROBLEMS

Name : ______________________ **Index Number** : __________

Country : ______________________

17th International Mathematics and Science Olympiad

Indonesia

21 January 2021

Instructions:

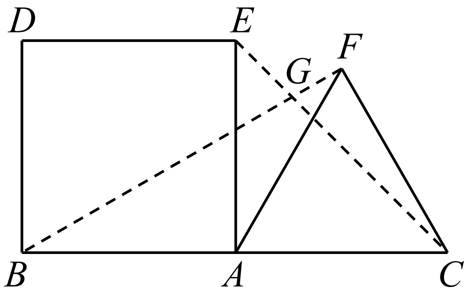
1. Write your name, country and index number on both the Question Booklet and the Answer Sheet.
2. Write your Arabic Numerical answers or English alphabet answers only in the Answer Sheet.
3. There are 25 questions in this paper.
4. For problems involving more than one answer, marks are only awarded when ALL answers are correct.
5. Each question is worth 1 mark. There is no penalty for a wrong answer.
6. You have 60 minutes to complete this paper.
7. Use black pen or blue pen or pencil to write your answer.

Do not turn over this page until you are told to do so.

SHORT ANSWER PROBLEMS

1. In year 2021, Ada's little brother is 10 years old. If Ada's age in 2021 is the sum of all the digits of the year she was born in. How old is she in 2021?

2. Points A, B, and C are on a straight line such that $AB = AC = 1$. Square $ABDE$ and equilateral triangle ACF are drawn on the same side of line BC and lines EC and BF intersect at G. What is the measure, in degrees, of angle EGB?



3. If the twelve-digit number $\overline{155a710b4c16}$ is divisible by 396, where a, b and c are three distinct digits. What is the value of $a+b+c$?

4. Each cell of the 3×4 grid below contains a positive integer less than 9 such that the sum of the four numbers in each row are all equal, while the sum of the three numbers in each column are all equal as well. What number must be placed in the gray cell?

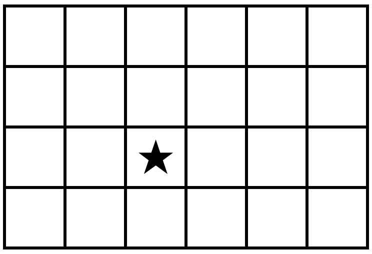
5	4		2
		3	6
3	5		

5. A computer store sells four types of laptop (A, B, C and D) and have sold a total of 2021 units. It is known that 20 type A laptops cost as much as 7 type B laptops, 4 type A laptops cost as much as 1 type C laptop and 4 type A laptops costs as much as 3 type D laptops. If the same amount was collected from each type, how many type A laptops were sold?

6. In the expression $(e\times7)\oplus f=19$, it is known that "$\oplus$" can be any one of the following operations: $+$, $-$, $\times$ or $\div$ and e and f are both one-digit non-zero positive integers. What is the sum of all possible values of $(e+f)$?

7. The first 999 positive integers are listed on a board. In each turn, Peter selects any two numbers from the list and replace it with their positive difference. He keeps on doing this until only one number is left. What is the largest possible value of this number?

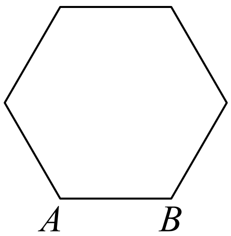
8. In the diagram below, how many different squares can be formed which contains the unit square with the "★" inside it?



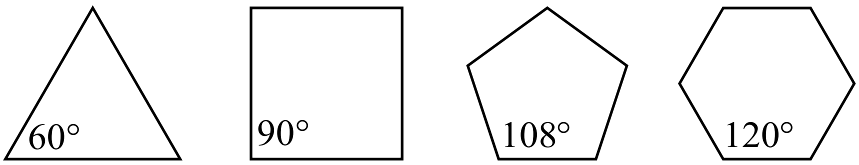
9. Cammy listed down all possible distinct five-digit positive even integers that can be formed using each of the digits 1, 3, 4, 5 and 9 exactly once. What is the sum of all the integers on Cammy's list?

10. Calculate the product of all the positive divisors of 120: $1\times2\times3\times4\times...\times120$. It is known that this product is divisible by some 2^k where k is an integer. What is the largest possible value of k?

11. In the diagram below, Andy used a metal rod to form a regular hexagon where the length of side AB is equal to 16 cm. He then divides the hexagon into two equal parts where one part is used to construct a square, while the other part is used to construct an isosceles triangle with a base of 18 cm. What is the sum of the areas, in cm², of the newly created square and triangle?

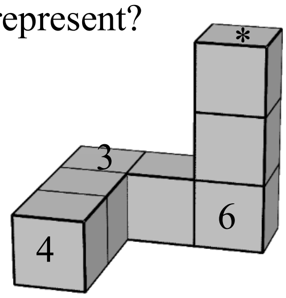


12. A regular triangle, a regular quadrilateral, a regular pentagon and a regular hexagon each have interior angles measured in integer degrees.



In total, how many kinds of regular polygons have interior angles measured in integer degrees?

13. The numbers 1 to 6 are printed on the six faces in each of the seven cubes and it is known that the sum of the two integers printed on any two opposite faces is 7. Arrange the seven cubes as shown in the diagram below such that the sum of the integers on any two faces touching each other is 8. What number does the "*" on the face of the top-most cube represent?

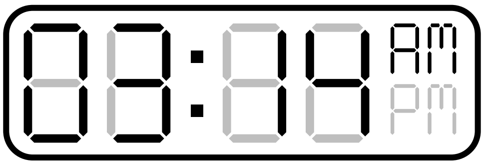


14. Seven points divide a circle into seven equal arcs. How many obtuse triangles can be formed by selecting three of these seven points as its vertices?

15. Positive integers from 1 to 45 inclusive are divided into 5 groups having 9 number each. What is the largest possible average of the medians of these 5 groups?
(Note: The median of a finite set of numbers is the "middle" number, when the numbers are listed from smallest to greatest or vice versa.)

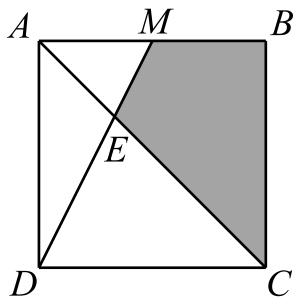
16. Find the smallest possible positive integer that is divisible by 18 which also has 18 different divisors?

17. I have a digital clock like the diagram shown below. During the 12-hour duration from 03:00 AM to 2:59 PM, how many minutes does the number "3" appear at least once?

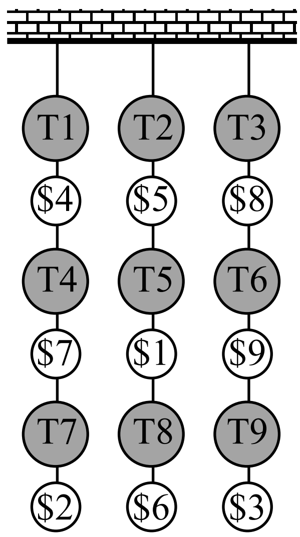


18. There are 32 coins such that all of them have a different weight. At least how many weighings on a standard 2-pan balance are needed to be able to determine both the heaviest and the second heaviest coins?

19. In the diagram below, *ABCD* is a square where *M* is the midpoint of *AB*. If line *AC* intersects *DM* at point *E* and the area of quadrilateral *BCEM* is 400 cm^2, then what is the area, in cm^2, of square *ABCD*?



20. In a target-shooting gallery, nine prizes ranging from \$1 to \$9, are placed and hanged up on three strings, as shown in the diagram below. The objective of the game is to hit the targets (indicated by the shaded circles) above each prize. If a target is hit, everything below it will be won by the player (and all the targets and prizes below becomes unavailable for the next player). It is known that the order of play was Alice went first, followed by Brian and then Colin. If each of them hits two targets and in total, Alice got \$18, Brian \$13 and Colin \$14, which target(s) were hit by Colin?



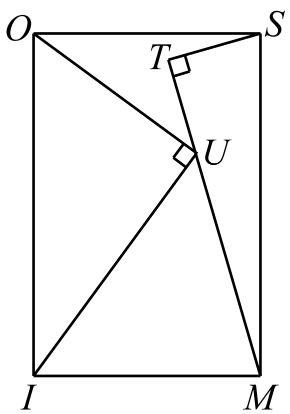
21. The following chart shows the answers given by Aaron, Betty, Clara and David in a ten-question true-or-false test. It is known that Aaron has eight, Clara seven and Betty only two correct answers. How many correct answers does David have?

Questions	1	2	3	4	5	6	7	8	9	10
Aaron	T	F	F	T	F	F	T	T	F	T
Betty	F	T	F	F	F	T	F	T	F	F
Clara	T	F	T	T	T	F	T	T	T	T
David	F	F	T	F	T	F	F	F	T	F

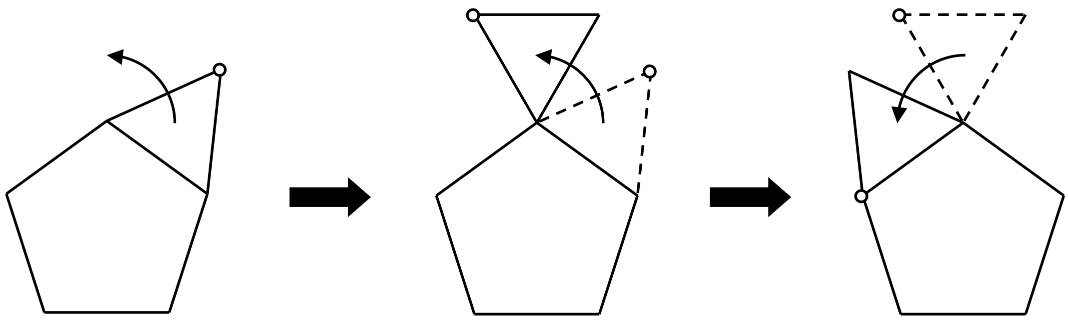
22. A group of 101 students went out for a school trip. Each of them visited at least one but at most two places out of the four places that were planned to be visited that day. Let N be the maximum number of students that visited the exact same places on the trip (e.g. they all visited only one place and this was the same place or they visited exactly two of the same places). What is the minimum value of N?

23. The average of $\overline{abc}$, $\overline{ab}$, $\overline{bc}$ and $\overline{ca}$ is M. If we remove $\overline{ab}$, then the average of the three remaining numbers will become N. If it is known that the difference between N and M is 40, then what is the maximum possible value for $\overline{abc}$?

24. In the diagram below, STM and OUI are right triangles inside rectangle $IMSO$ such that point U lies on line TM. If $ST = 21$ cm, $TM = 72$ cm and $OU = 45$ cm, what is the area, in cm^2, of rectangle $IMSO$?



25. An equilateral triangle rotates around a regular pentagon of side 21 cm as shown in the diagram below.



Original position

What is the length, in cm, of the trace made by the white point until it first returns to its original position? (Let $\pi = \frac{22}{7}$)

ESSAY PROBLEMS

Name: ______________________ Country: ____________ Index Number: _______

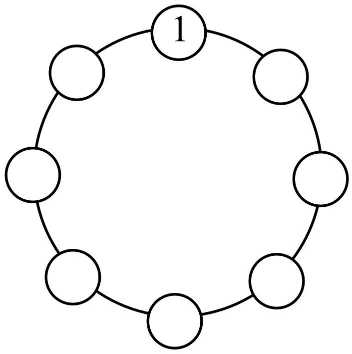
1. Mali tries to insert some number of " + " signs between the digits of 777777777777 (twelve copies of 7s) so that the resulting expression is a multiple of 30. How many ways can she do it?

Answer : ______________________ ways

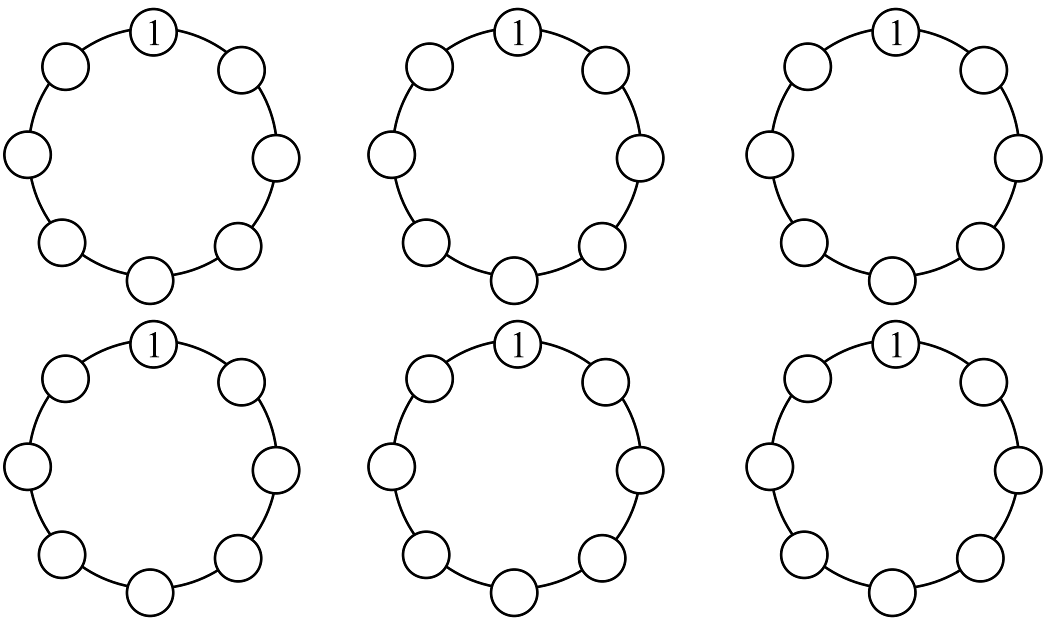
ESSAY PROBLEMS

Name: ______________________ Country: ____________ Index Number: _______

2. Place each of the numbers 2, 3, 4, 5, 6, 7 and 8 exactly once in the eight empty circles (the position of 1 is already fixed) such that the sum of each pair of adjacent numbers is a prime number. Note that the diagrams are considered the same if it can be obtained by reflection or rotation. List down all possible diagrams.



Answer :



ESSAY PROBLEMS

Name: ______________________ Country: ____________ Index Number: _______

3. Four runners (A, B, C and D) participated in a race. Before the event, all four of them (I, II, III and IV) anonymously predicted the final rankings as shown in the table below:

	1st	2nd	3rd	4th
I's prediction	B	C	A	D
II's prediction	C	A	D	B
III's prediction	D	B	A	C
IV's prediction	B	C	D	A

If I and III got 2 correct predictions each while II and IV got 1 correct prediction each. Determine the final placings of the four participants.

Answer :

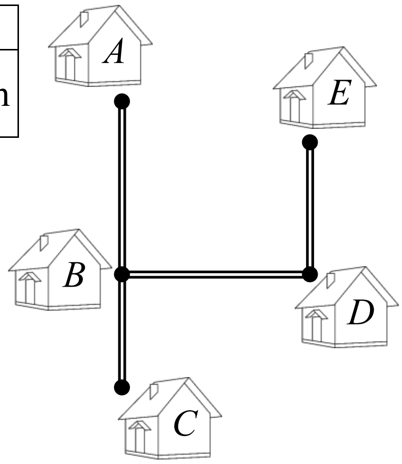
1st	2nd	3rd	4th

ESSAY PROBLEMS

Name: ______________________ Country: ____________ Index Number: _______

4. As shown in the diagram below, 5 persons (A, B, C, D and E) live in one village. The table below shows the sum of the distance travelled by the other 4 people when going to a one house. If a party is to be held in E's house, what is the total distance travelled, in meters, of the 4 people (A, B, C and D)?

Party house	*A*	*B*	*C*	*D*
Total distance travelled by 4 people	4180 m	2530 m	3610 m	3130 m



Answer : ______________________ m

ESSAY PROBLEMS

Name: ______________________ Country: ____________ Index Number: _______

5. Three motorcyclists started their rides at the same time. During the whole journey, they travelled for some time, took a rest for some time and then travelled to return to their homes at the same time. It's known that the first motorcyclist travelled twice as much as the second motorcyclist rested, the second motorcyclist travelled three times as much as the third motorcyclist rested and the third motorcyclist travelled four times as much as the first motorcyclist rested. If the sum of the speeds of the first and the third is 48 km/h, find the speed of the second one.

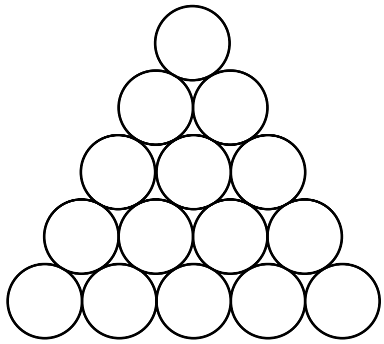
Note: Each motorcyclist travelled at the same speed for the whole journey.

Answer : ____________________ km/h

ESSAY PROBLEMS

Name: ______________________ Country: ____________ Index Number: _______

6. A pair of circles are called “neighbors” if they touch each other. How many pairs of “neighbors” are there in the diagram below?

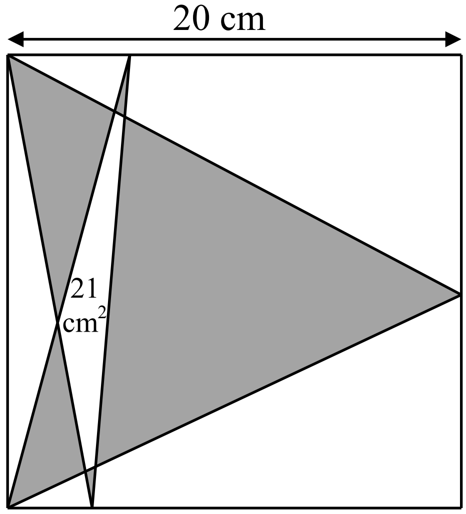


Answer : ______________________ pairs

ESSAY PROBLEMS

Name: ____________________ Country: ___________ Index Number: ______

7. In the diagram below, the square has side length 20 cm, and the area of the 'middle pentagon' inside the square is 21 cm^2. What is total area, in cm^2, of all the shaded regions?



Answer : ______________________ cm^2

ESSAY PROBLEMS

Name: ______________________ Country: ____________ Index Number: _______

8. In a chess tournament, each participant plays with every other participant exactly once. Each participant gets 1 point for a win, 0.5 point for a draw and 0 points for a loss. At most how many of the 40 participants can score 24 points or more?

Answer : ____________________ participants

ESSAY PROBLEMS

Name: ______________________ Country: ____________ Index Number: _______

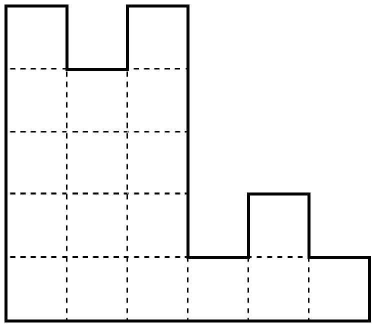
9. There are 8 consecutive non-zero positive integers such that their sum can be expressed as the sum of 7 consecutive positive integers and it cannot be expressed as the sum of 3 consecutive positive integers. What is the minimum value of the largest number among the 8 consecutive positive integers?

Answer : ___________________________________

ESSAY PROBLEMS

Name: ____________________ Country: ____________ Index Number: ________

10. Dissect the diagram below into two congruent parts which may be rotated or reflected. Find the perimeter, in cm, of each part if the area of the original diagram is 72 cm^2.



Answer : ______________________ cm

ESSAY PROBLEMS

Name: ______________________ Country: ____________ Index Number: _______

11. Five teams took park in a soccer tournament, where each team plays each other exactly once. Each team gets 3 points for a win, 1 point for a draw and 0 points for a loss. After all matches has been played, is it possible that the total point scored of these five teams are five consecutive positive integers? If yes, provide an example.

example

v.s.	I	II	III	IV	V	points
I	■					
II		■				
III			■			
IV				■		
V					■	

☐ My answer is **YES**,

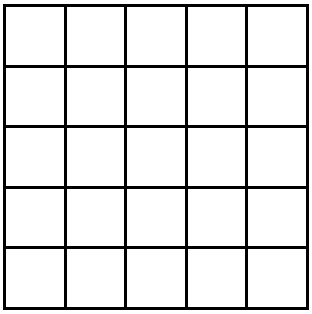
☐ My answer is **NO**, the reason is

Answer :

ESSAY PROBLEMS

Name: ______________________ Country: ____________ Index Number: _______

12. In each cell of the 5×5 grid shown below, Peter may place several stones or leave it empty such that the total number of stones for each and every row and columns are all different. What is the minimum number of stones that Peter must use to satisfy the conditions?

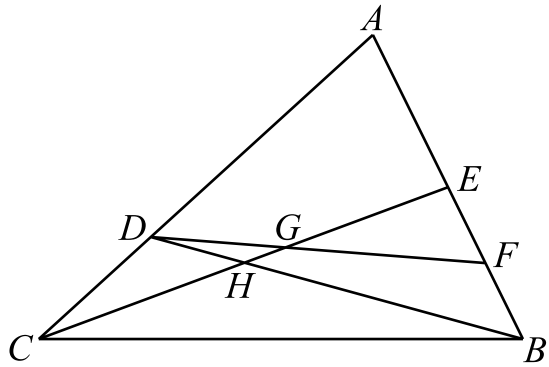


Answer : ____________________ stones

ESSAY PROBLEMS

Name: ____________________ Country: ____________ Index Number: ________

13. In the diagram below, the area of triangle EFG is equal to the area of triangle CDG, the ratio between the area of triangles EBC and BCD is 3 : 2 and $AD : DC = 2 : 1$. If the area of triangle BDF be 29 cm^2, then what is the area, in cm^2, of triangle ABC?

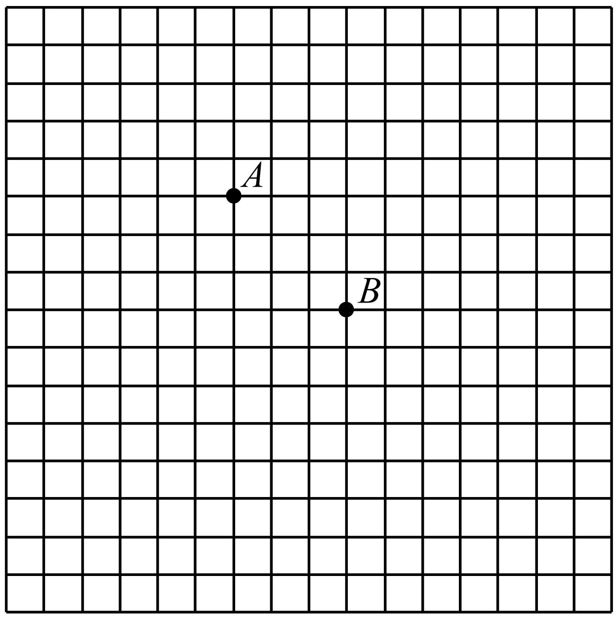


Answer : ____________________ cm^2

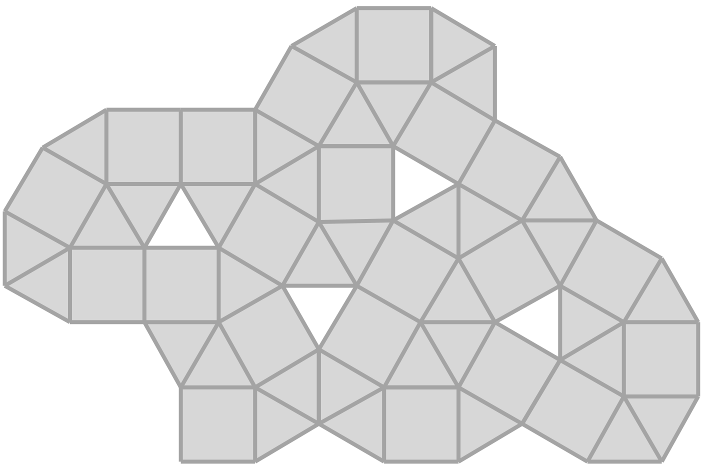
17th International Mathematics and Science Olympiad

EXPLORATION PROBLEMS

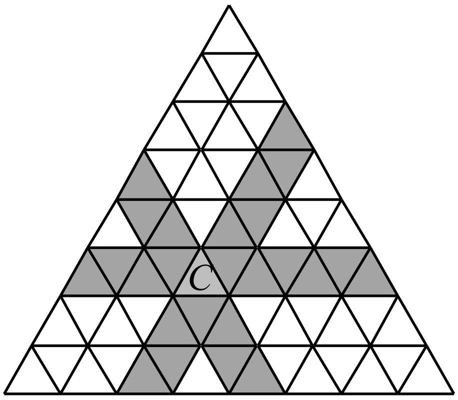
1. Suppose that we are living in a city where the streets are laid out on a square grid as shown below. In this city, you are only allowed to move along vertical lines (streets that are running in the north–south direction) or along horizontal lines (streets that are running in the east–west direction). Suppose your home is located in point A and you wanted to go to your friend's house (point B), in order to measure the distance travelled, you can't just get a ruler and measure the distance between the two points but rather as the distance you have to walk along the streets. Therefore, we define the *TCG distance* between A and B and this can be obtained by going along one of its shortest routes. For example, from A to B – 3 right and 3 down or 3 down and 3 right, so the *TCG distance* between A and B in the diagram is 6.
   (a) How many points in the grid have *TCG distance* to A and to B are both 6? Draw the diagram. (2 marks)
   (b) How many points in the grid have *TCG distance* to A and to B are both 12? Draw the diagram. (4 marks)



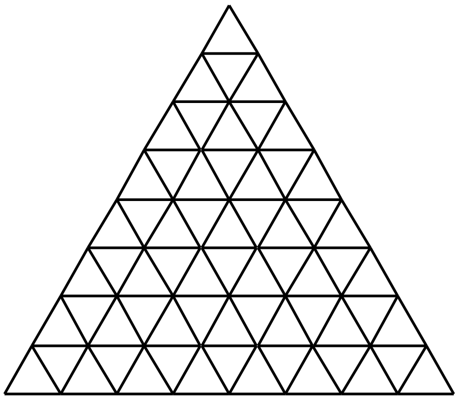
2. Divide the diagram below (only along the lines) such that the resulting diagram will be divided into 7 identical (included its reflection) pieces.



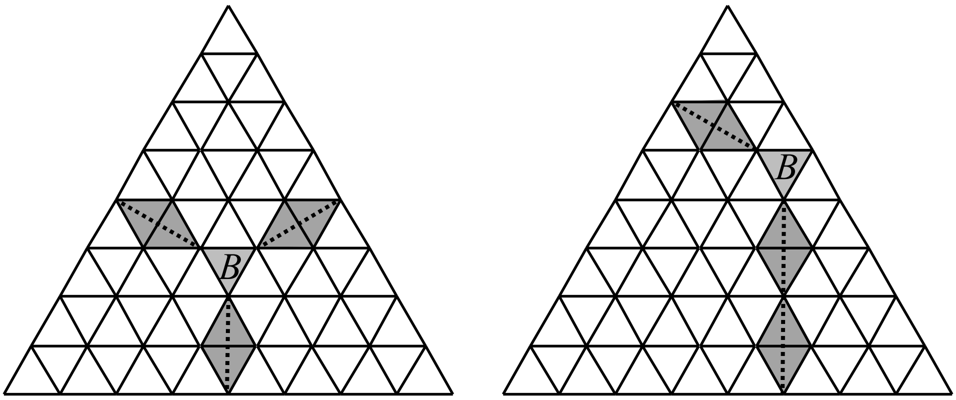
3. The diagram below shows a triangular chessboard. A chess piece *castle*, indicated as C, may attack along any of the three directions as highlighted below.



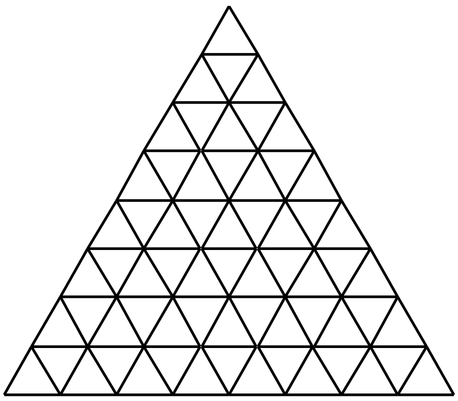
(a) Place as many *castles* in the chessboard as possible so that no two *castles* could attack each other. (3 marks)



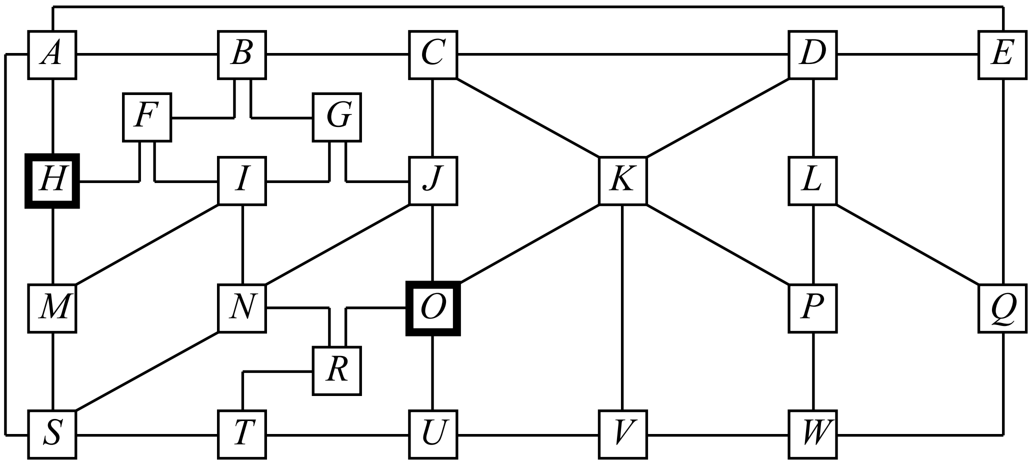
(b) A chess piece *bishop,* indicated as B, may attack along the direction of any of its angles. The two diagrams below show two examples on where the attack ranges of *bishops* are highlighted. (While the broken lines indicate the direction of the angles.)



Place as many *bishops* in the chessboard as possible so that no *bishop* is in a position that is attacking another. (3 marks)



4. The diagram below shows a map of 23 towns connected by roads, with a video arcade in each town. Andy, who loves to play video games, works in an office in town *O*. Every day after work, he visits all the towns exactly once to play before reaching home which is located in town *H*, spending much of his free time at the video arcades, which means that he is ignoring his girlfriend. One day, she decides to meet him on his way home, but he varies his routes all the time. However, she has discovered that there is one road which he must always pass in all the possible routes he can use, so she plans to wait for him along it. Which road is it?



5. Anna and Boris play a game using the integers 1, 2, 3, 4, 5, 6, 7 and 8. Anna first selects a positive integer $k \le 8$, then Boris chooses *k* of these eight numbers, then Anna chooses three of among the eight numbers. If the sum of every pair of numbers chosen by Anna is among the numbers chosen by Boris, then Anna wins.
   (a) What is the minimum value of *k* for which Anna has a winning strategy? (2 marks)
   (b) If Anna has a winning strategy for *k*, give a counterexample to show that for $k-1$ Anna cannot guarantee a win. (4 marks)

6. There are 100 positive integers whose sum is equal to their product. Find all possible products.

MATHEMATICS
EXPLORATION PROBLEMS
ANSWER SHEET

Name : ______________________________ **Index Number** : ___________

Country : ______________________________

17th International Mathematics and Science Olympiad

Indonesia

22 January 2021

The following table is for jury use only.

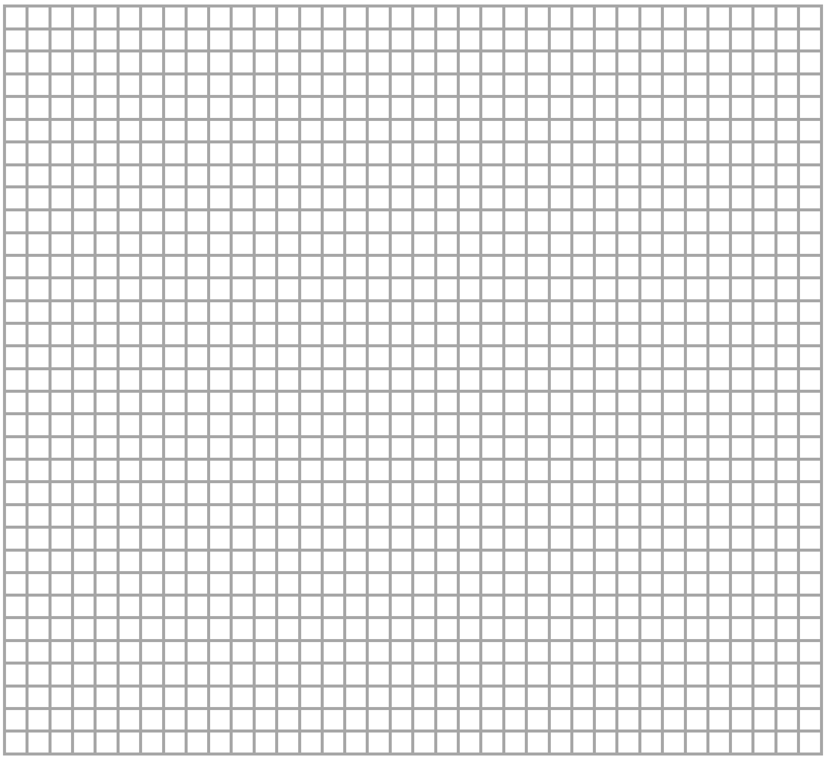
No.	1	2	3	4	5	6	*Total*
Score							
Signature							
Score							
Signature							

EXPLORATION PROBLEMS
ANSWER SHEET

Country: ____________ Name: ____________________ Index Number : ______

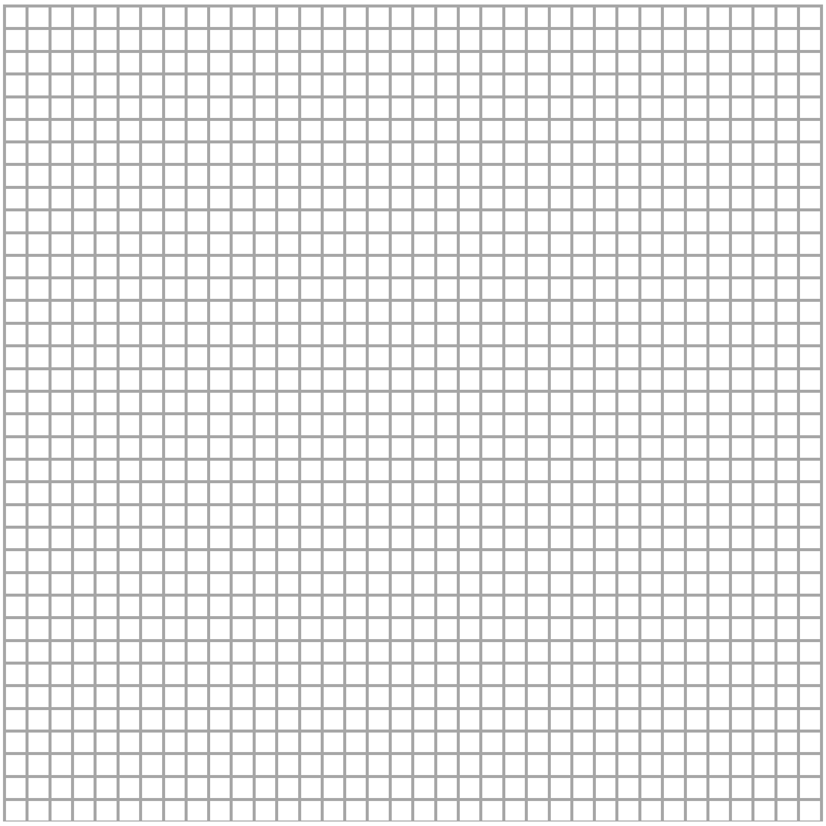
Score
(a)
(b)

(1) (a) There are _________________ points as shown in the diagram below :



(no partial score.)

(b) There are _________________ points as shown in the diagram below:



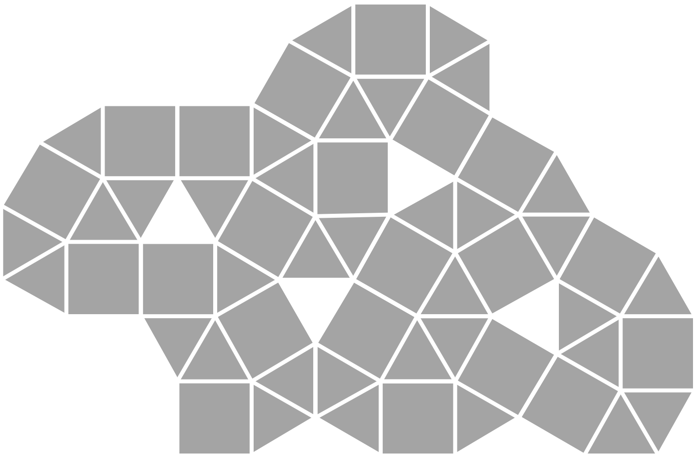
(no partial score.)

EXPLORATION PROBLEMS
ANSWER SHEET

Country: ____________ Name: ____________________ Index Number : ______

(2)

Score

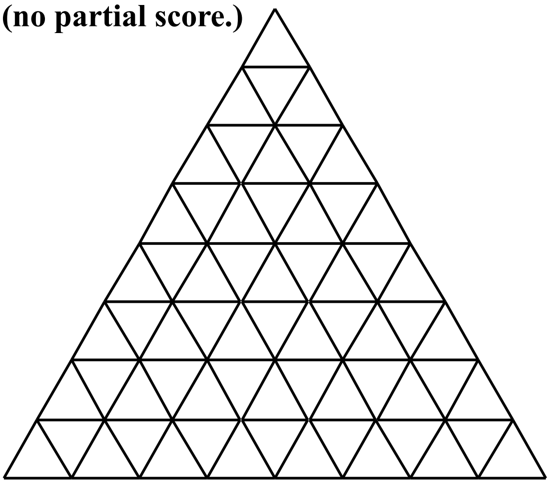


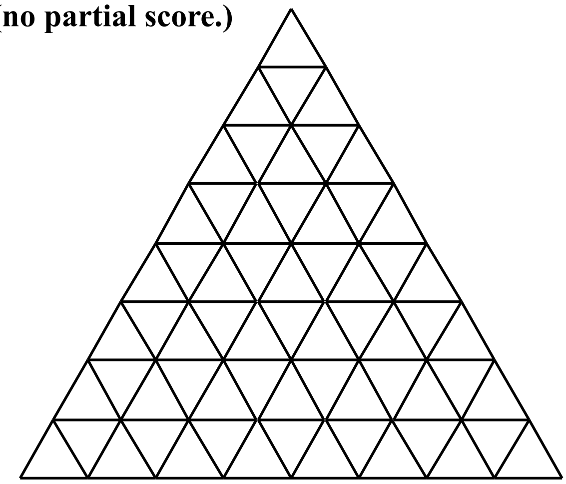
(no partial score.)

(3) (a) There are __________ castles as shown in the diagram below: **(no partial score.)**

(b) There are __________ bishops as shown in the diagram below: **(no partial score.)**

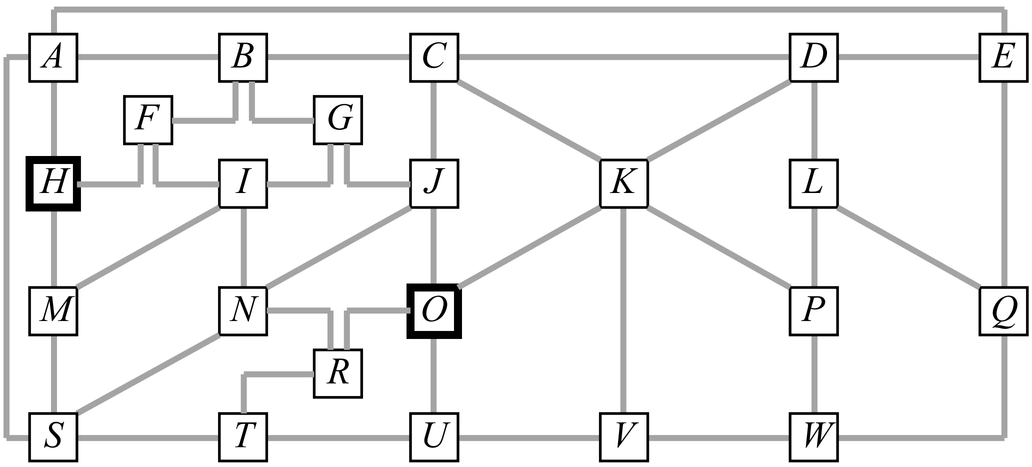
Score
(a)
(b)





(4) Draw a possible route and she can wait for him along the road between towns ______ and ______. **(no partial score.)**

Score



EXPLORATION PROBLEMS
ANSWER SHEET

Country: ______________ Name: ________________________ Index Number : ________

(5) (a) The minimum value of k for which Anna has a winning strategy is ______.

(no partial score.)

Score
(a)
(b)

(b) If Anna has a winning strategy for $k =$ ______, then Boris can choose $k-1$ numbers such as ____, so that Anna cannot guarantee a win.

(no partial score.)

(6) The possible products are:

Score

______________________________, ______________________________,
______________________________, ______________________________,
______________________________, ______________________________,
______________________________, ______________________________,
______________________________, ______________________________,
______________________________, ______________________________,
______________________________, ______________________________,
______________________________, ______________________________,
______________________________, ______________________________,
______________________________, ______________________________,
______________________________, ______________________________,
______________________________, ______________________________,
______________________________, ______________________________.

(The number of correct answer minus the number of incorrect answer is net number correct answer.)

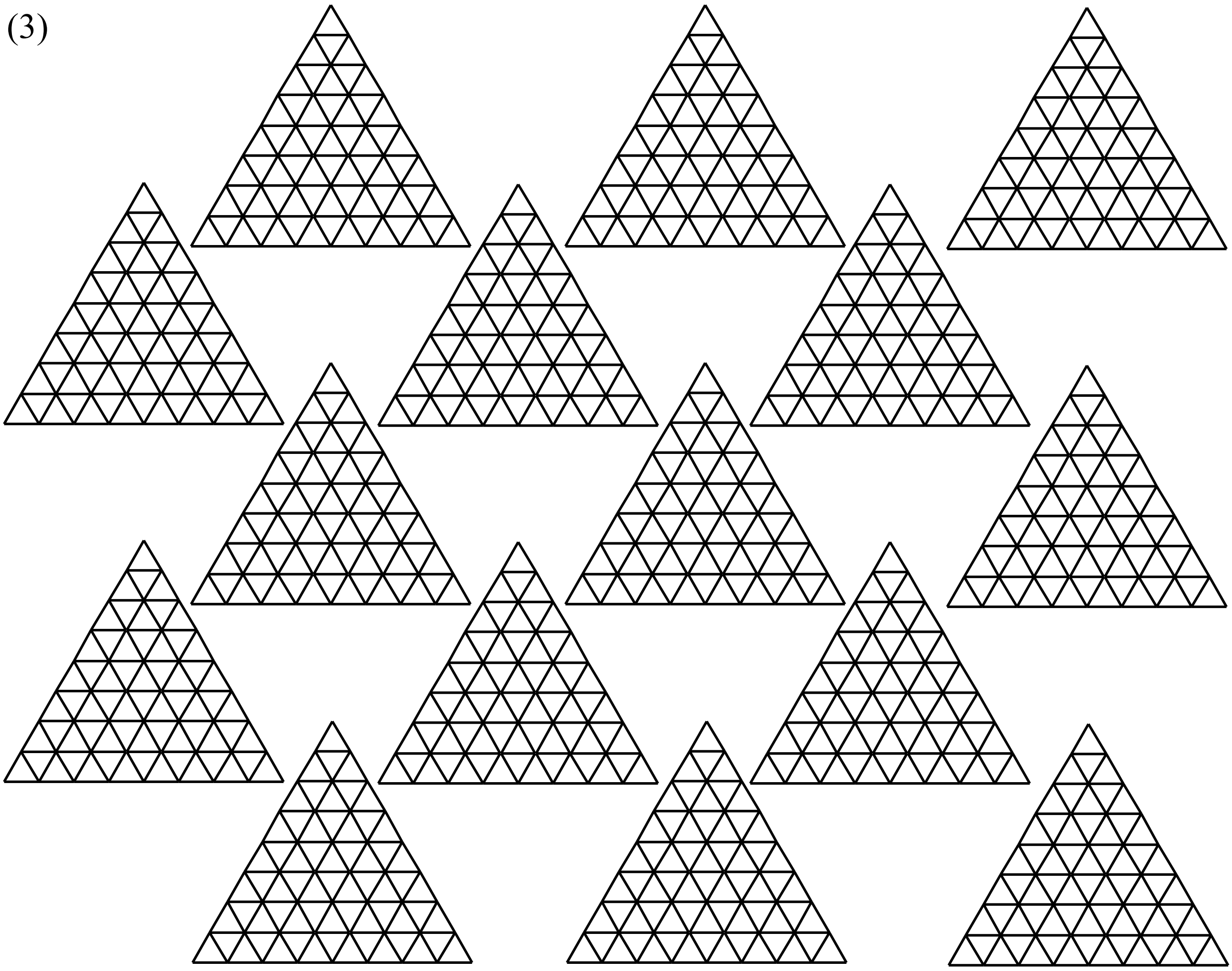
For Sketch Use

(1)

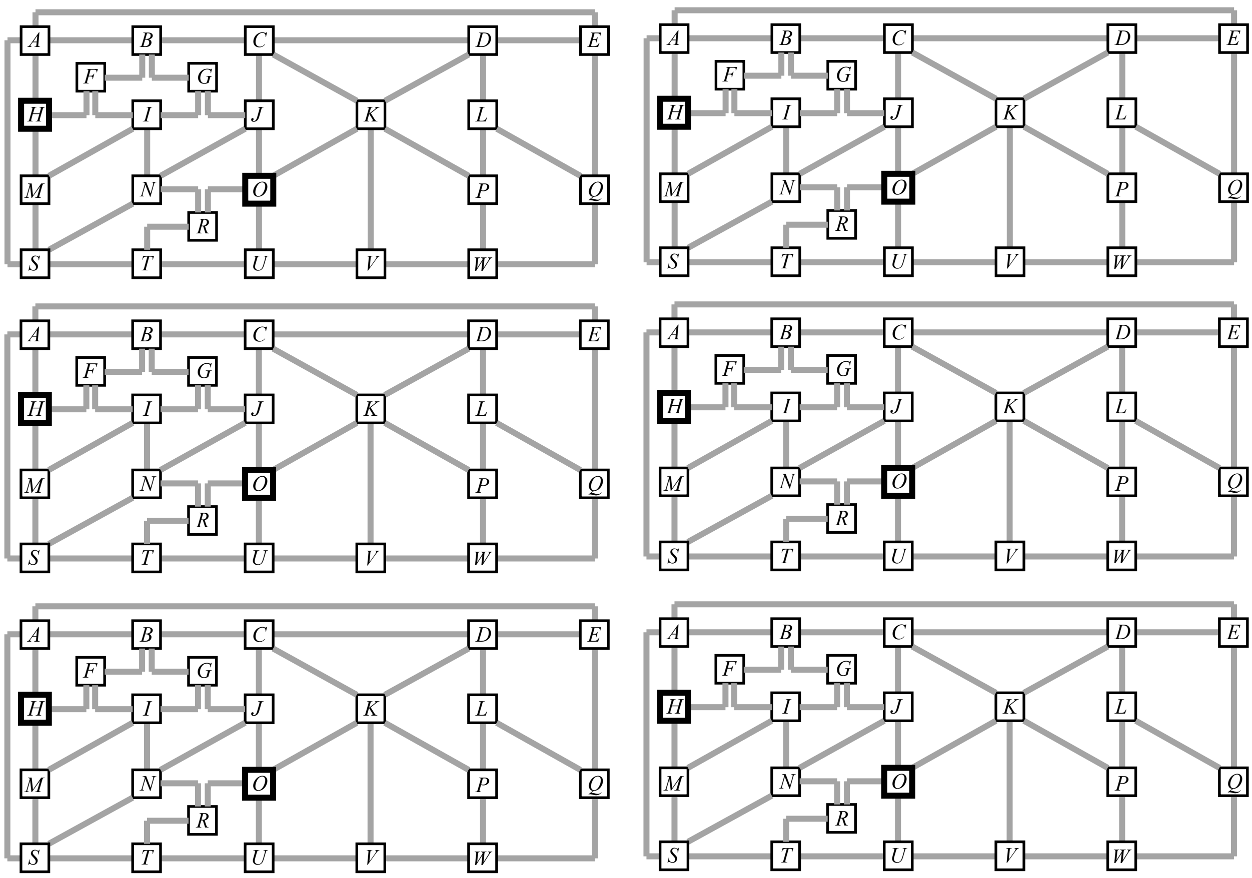
(2)

For Sketch Use

(3)



(4)



17th IMSO Answers

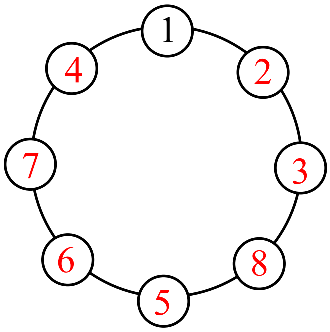
SHORT ANSWER

1.	25	**2.**	75°	**3.**	6	**4.**	4	**5.**	860
6.	25	**7.**	998	**8.**	14	**9.**	1199976	**10.**	24
11.	252	**12.**	22	**13.**	3	**14.**	21	**15.**	31
16.	180	**17.**	225	**18.**	35	**19.**	960	**20.**	T2 and T3
21.	4	**22.**	11	**23.**	519	**24.**	3750	**25.**	704

ESSAY PROBLEMS

1. 55

2.



3.

1st	2nd	3rd	4th
D	C	A	B

4. 4390

5. 32

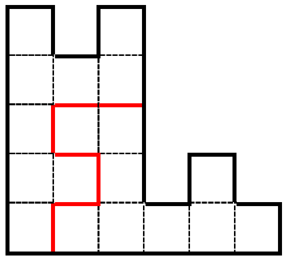
6. 30

7. 158

8. 31

9. 21

10.



, 40

11. Yes, the final points are 3, 4, 5, 6 and 7 points and one example is as following:

	I	II	III	IV	V	points
I		1	3	0	3	7
II	1		1	3	1	6
III	0	1		3	1	5
IV	3	0	0		1	4
V	0	1	1	1		3

12. 23, the diagram below shows one way to fill the grid.

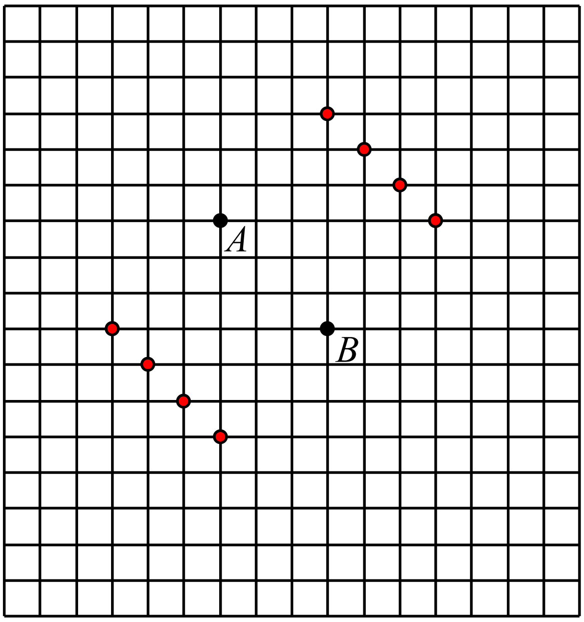
0	0	0	0	0	**0**
0	0	0	0	1	**1**
0	0	0	0	5	**5**
0	0	0	6	1	**7**
2	3	4	0	1	**10**
2	**3**	**4**	**6**	**8**	

13. 174

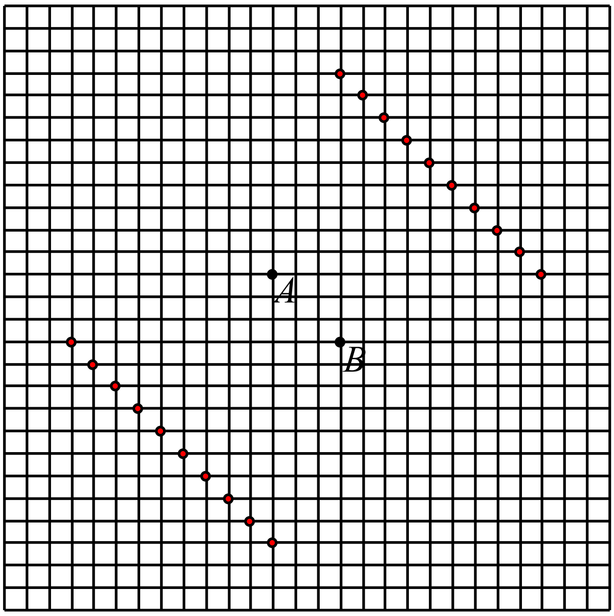
EXPLORATION PROBLEMS

1.

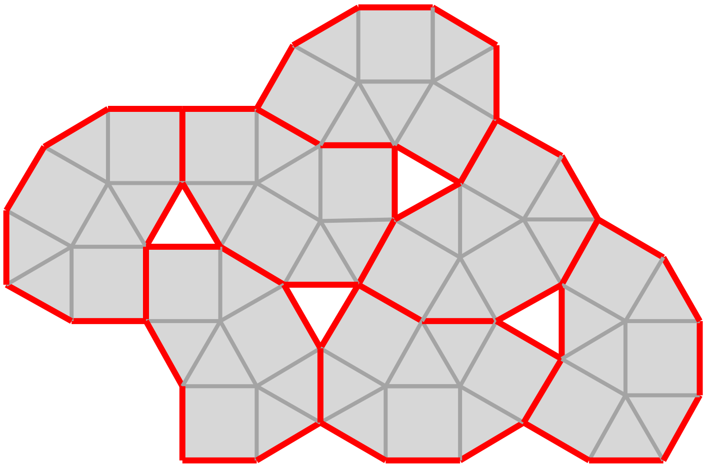
(a) There are 8 points as shown in the diagram below:



(b) There are 20 points as shown in the diagram below:

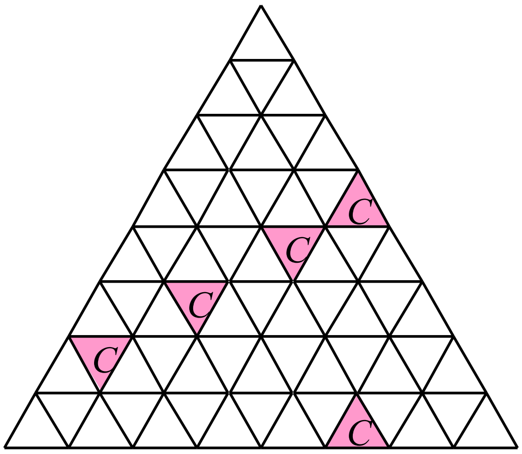


2.

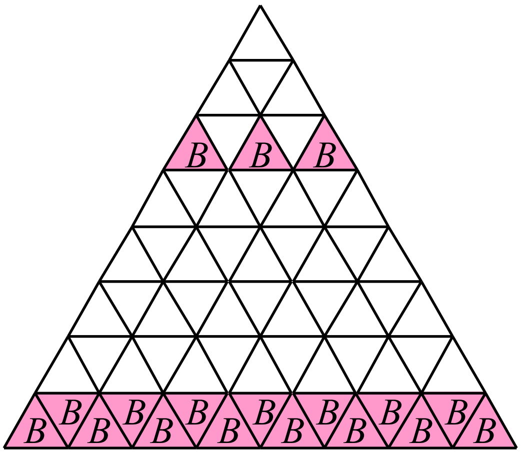


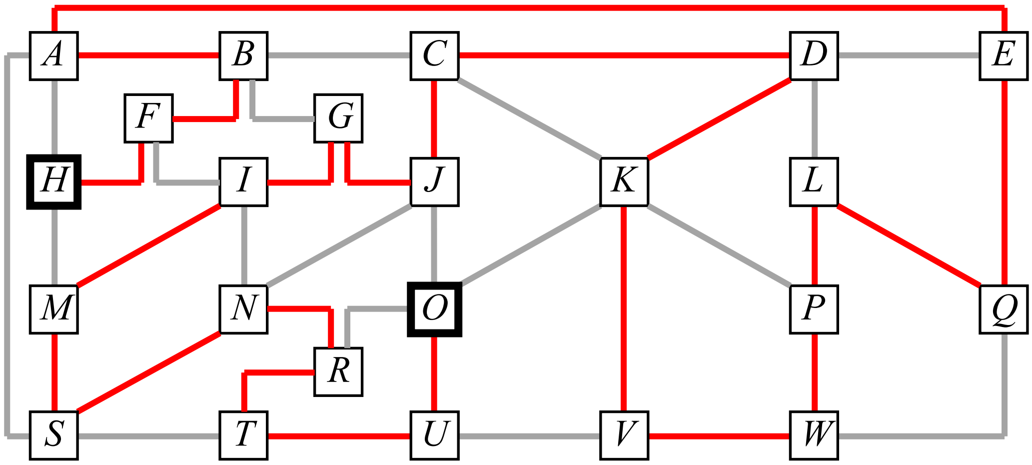
3.

(a) 5 castles are placed as shown in the diagram below:



(b) 18 bishops are placed as shown in the diagram below:



4.	She can wait for him along the road between towns C and D. One possible route as shown in the diagram below: 	
5.	**(a)**	The minimum value of k for which Anna has a winning strategy is 6.
	(b)	If Anna has a winning strategy for $k=6$, then Boris can choose $k-1$ numbers such as 1, 2, 3, 5 and 7, so that Anna cannot guarantee a win.
6.	There are 5 possible products: $$2+100+\underbrace{1+1+\ldots+1}_{98\text{ terms}}=200=2\times100\times\underbrace{1\times1\times\ldots\times1}_{98\text{ terms}}$$ $$4+34+\underbrace{1+1+\ldots+1}_{98\text{ terms}}=136=4\times34\times\underbrace{1\times1\times\ldots\times1}_{98\text{ terms}}$$ $$10+12+\underbrace{1+1+\ldots+1}_{98\text{ terms}}=120=10\times12\times\underbrace{1\times1\times\ldots\times1}_{98\text{ terms}}$$ $$4+4+7+\underbrace{1+1+\ldots+1}_{97\text{ terms}}=112=4\times4\times7\times\underbrace{1\times1\times\ldots\times1}_{97\text{ terms}}$$ $$2+2+3+3+3+\underbrace{1+1+\ldots+1}_{95\text{ terms}}=108=2\times2\times3\times3\times3\times\underbrace{1\times1\times\ldots\times1}_{95\text{ terms}}$$	