

---

# MATHEMATICS

## SHORT ANSWER PROBLEMS

---

Name : \_\_\_\_\_

Participant ID : \_\_\_\_\_

Country : \_\_\_\_\_



**21<sup>st</sup> International Mathematics and Science Olympiad**  
**Wenzhou, China**  
**03 October 2024**

**Instructions:**

1. Write your name, country and participant ID on both the Question Booklet and the Answer Sheet.
2. Write your Arabic Numerical answers or English alphabet answers only in the Answer Sheet.
3. There are 25 questions in this paper.
4. For problems involving more than one answer, marks are only awarded when ALL answers are correct.
5. Each question is worth 1 mark. There is no penalty for a wrong answer.
6. You have 60 minutes to complete this paper.
7. Use black pen or blue pen or pencil to write your answer.

---

**Do not turn over this page until you are told to do so.**

# SHORT ANSWER PROBLEMS

- The characters from the string “21IMSO2024” is to be written consecutively up until the 2024<sup>th</sup> character.

$$\underbrace{21IMSO202421IMSO2024 \dots}_{2024 \text{ characters}}$$

How many times will the digit “2” appear in the sequence of characters?

- The “IMSO Robot” is controlled by a gamepad with buttons  $\uparrow$   $\downarrow$   $\Rightarrow$  and  $\Leftarrow$ . When  $\uparrow$  is pressed 1 time, the “IMSO Robot” will go 1 unit to the north,  $\downarrow$  is pressed 1 time, the “IMSO Robot” will go 1 unit to the south,  $\Rightarrow$  is pressed 1 time, the “IMSO Robot” will go 1 unit to the east,  $\Leftarrow$  is pressed 1 time, the “IMSO Robot” will go 1 unit to the west.

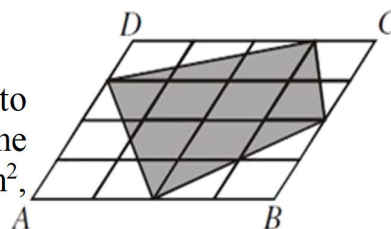
Anna controls the “IMSO Robot” from  $A$  to  $B$  by pressing the buttons as follows:

$\uparrow \Rightarrow \Rightarrow \uparrow \Leftarrow \uparrow \Rightarrow \Rightarrow \uparrow \Rightarrow \downarrow \Leftarrow \uparrow \uparrow \Leftarrow \downarrow \Rightarrow \Rightarrow \Rightarrow \uparrow \Leftarrow \uparrow \uparrow \Rightarrow \downarrow \Leftarrow \downarrow$

Now, Bob wants to control the “IMSO Robot” from  $B$  to  $A$ . What is the least number of times that Bob has to press the buttons?

- Annie is 22 years younger than Betty and Cindy is 6 years younger than Betty. The age difference between Diana and Elsa is 29 years. Diana is twice as old as Annie, and the sum of their ages is 54. Elsa is not older than Cindy. What is the sum of the ages of Betty, Cindy, and Elsa?

- In the diagram, the parallelogram  $ABCD$  is divided into 16 identical small parallelograms. The area of the shaded quadrilateral is  $68 \text{ cm}^2$ . What is the area, in  $\text{cm}^2$ , of the parallelogram  $ABCD$ ?



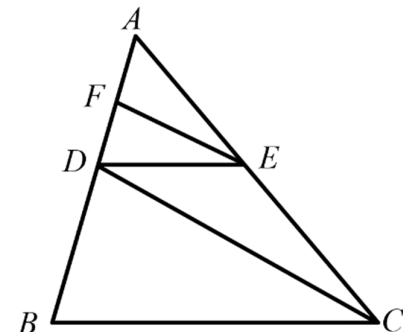
- Yuan bought mangoes and melons at the market. The price of a mango is  $\frac{3}{7}$  of the price of a melon. At first, Yuan used  $\frac{3}{8}$  of his money to buy 6 melons and 2 mangoes. The following week, he used the rest of his money to buy 2 melons and some mangoes. How many mangoes did Yuan buy in the second week?

6. In a TV show, four women (Ana, Bianca, Celine and Diana) and four men (Andrew, Ben, Chris and Daniel) are to be divided into four groups. Each group consists of a man and a woman. If Ana cannot be grouped with Ben, in how many different ways can we group them?
7. Let  $M$  be a two-digit positive integer such that the greatest common divisor of  $M + 9$ ,  $M + 35$ , and  $M + 74$  is greater than 1. What is the sum of all possible values of  $M$ ?
8. Let  $a$  be a positive factor of 12 and  $b$  be a positive fraction where  $a > b$  and  $a + b = a \times b$ . What is the product of all possible values of  $b$ ?
9. Fifty students take a test with four questions. The numbers of students who correctly answer the questions are given in the following table:

| Question | Number of students |
|----------|--------------------|
| Q1       | 48                 |
| Q2       | 42                 |
| Q3       | 45                 |
| Q4       | 15                 |

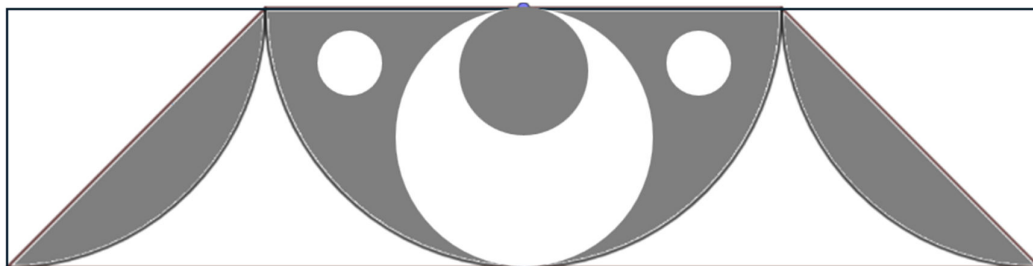
If no student can solve all questions, what is the largest possible number of students who solve both Q3 and Q4 correctly?

10. In the diagram,  $ABC$  is a triangle.  
 $DE$  is parallel to  $BC$  and  $FE$  is parallel to  $DC$ .  
 If  $AF = 4$  cm and  $FD = 6$  cm,  
 what is the length, in cm, of  $DB$ ?



11. The sum of two three-digit numbers  $\overline{7A2}$  and  $\overline{B38}$  is divisible by 18. What is the largest possible product of  $A$  and  $B$ ?
12. A team participating in the IMISO 2024 competition consists of six male and six female students. They will be staying in a dormitory consisting of six rooms in a row. Each room must be occupied by exactly two students of the same gender. Moreover, any two adjacent rooms must be occupied by students of different genders. In how many ways can they make such arrangements?

13. Suppose that  $A$  is a 2024-digit number which is divisible by 225. What is the last three digits of the smallest possible number  $A$ ?
14. Inspector Daniel is investigating a crime. He apprehends three suspects: Anne, Bill, and Caroline. He knows that no one else could have participated in the crime. During the interrogation, each of the suspects make two statements:
- Anne : I didn't do it.  
Bill did it.
  - Bill : I didn't do it.  
Caroline did it.
  - Caroline : I didn't do it.  
Anne did it alone.
- Daniel also discovered that one of the suspects always tells the truth and the other two always give false statements. Who is the truth-teller?
15. Let  $n$ ,  $m$  and  $k$  be positive integers such that  $n + n \times m + n \times m \times k = 13$ . What is the sum of all possible value of  $k + k \times m + k \times m \times n$ ?
16. Xiaoming wants to buy school supplies. The total number of pens, pencils and erasers that he wants to buy is 100. The price of one pen is 6 yuan, one pencil is 4 yuan and one eraser is 1 yuan. He spends exactly 200 yuan and buys at least one of each type. How many different combinations of the number of pens, pencils and erasers can he buy?
17. Wei calculated the square and the cube of a positive integer. When she looked at the results, she noticed that all the digits from 1 to 8 had been used, each exactly once. What is the value of this positive integer?
18. In the diagram, the ratio of the width and the length of the rectangle is 1 : 4. The ratio of the radius of the semicircle to the radius of the biggest circle to the radius of the grey circle to the radius of the two smallest circles is 8 : 4 : 2 : 1. If the area of the rectangle is  $1024 \text{ cm}^2$ , What is the total area, in  $\text{cm}^2$ , of the shaded regions? (Use  $\pi = 3.14$ )



19. How many numbers between 2024 and 20240 which contain exactly two consecutive 9's are there?

For example: 2099 and 9929 are included, but not 9295 or 1999.

20. Huang took a test and answered all the questions. The test consists of questions from three subjects: Algebra, Geometry and Combinatorics.

It turned out that he correctly answered:

- 67.5% of the Algebra questions,
- 86% of the Geometry questions,
- 80% of the Combinatorics questions,
- 70% of the questions on the subjects of Algebra and Combinatorics and
- 85% of the questions on the subjects of Geometry and Combinatorics.

What is the percentage of correct answers from all the questions?

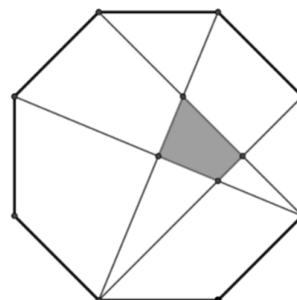
21. How many numbers with 5 distinct non-zero digits are there that are divisible by 3?
22. Each of the places between the numbers below must be filled with one of the operational signs:  $+$ ,  $-$ ,  $\times$ ,  $\div$  without repetition. No parentheses are allowed.

$$1\_2\_3\_4\_5 = X$$

Let  $\lfloor X \rfloor$  denote the greatest integer not exceeding  $X$ . For example:  $\lfloor 3.74 \rfloor = 3$ ,  $\lfloor -3.14 \rfloor = -4$ . What is the greatest possible value for  $\lfloor X \rfloor^2$ ?

23. In how many distinct ways can the letters in the word "SCIENCE" be arranged such that no two identical letters are adjacent?

24. In the diagram, the side length of the regular octagon is 20 cm. What is the area, in  $\text{cm}^2$ , of the shaded region?



25. What is the simplest form of the following fraction

$$\frac{1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + 2022 \times 2023 + 2023 \times 2024}{2024 \times 1 + 2023 \times 2 + 2022 \times 3 + \dots + 2 \times 2023 + 1 \times 2024}?$$

- End of Short Answer Problems -